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The stability of long steady three-dimensional salt fingers to long-wavelength perturbations

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The stability of three-dimensional salt fingers to three-dimensional perturbations is considered. It is found that when square fingers are subjected to long-wavelength perturbations, then the same types of instability exist as for two-dimensional salt fingers. There are quantitative changes to the growth rates and the positions of marginal stability.

1. Introduction

It is known from laboratory experiments that salt fingers have a square planform (Turner 1979). Previous work on the stability of salt fingers has considered the motion in two-dimensional (planar) salt fingers (Stern 1969; Holyer 1981, 1984). The aim of this paper is to establish whether or not the same instabilities exist for square salt fingers, and if they do, what the quantitative changes are.

For two-dimensional salt fingers subjected to long-wavelength internal-wave perturbations Holyer (1981) shows that the collective instability first occurs when

$$\frac{\beta F_S - \alpha F_T}{\nu(\alpha T_z - \beta S_z)} > \frac{1}{3}, \tag{1.1}$$

where F_T and F_S are the heat and salt fluxes through the salt fingers, ν is the kinematic viscosity and T_z and S_z are the heat and salt gradients. It is shown in this paper that for square salt fingers the collective instability still occurs, but first appears for a larger value of the stability ratio, namely when

$$\frac{\beta F_S - \alpha F_T}{\nu(\alpha T_z - \beta S_z)} > \frac{2}{3}. \tag{1.2}$$

The non-oscillatory instability found for two-dimensional fingers by Holyer (1984) also occurs for three-dimensional fingers, with a modified growth rate.

There is a short discussion in the conclusions about the relationship between the theory and laboratory observations, as well as suggestions for future theoretical, numerical and experimental work. For a discussion presented from the viewpoint of the experimentalist, McDougall & Taylor (1984) is the most recent and thorough reference. In §2 the equations for steady salt fingers are obtained in a form that allows two-dimensional and square salt fingers to be considered from the same equations, by varying a parameter. In §3 long-wavelength perturbations to this basic steady salt-finger state are considered, and the instabilities are obtained in §4. The structure needed to solve the equations is presented, but much of the detail is omitted since the algebra is extremely long and tedious. The results obtained agree with the two-dimensional results (Holyer 1984) in the appropriate limits and satisfy various symmetry checks. The determined reader is invited to check the algebra, but is warned of its length.

2. The salt fingers

We consider motion in an unbounded region of incompressible stratified fluid with coordinates chosen so that z is vertically upwards and x and y are in the horizontal plane. The temperature field $\tilde{T}$ and the salinity field $\tilde{S}$ are given by

$$\tilde{T} = T_z z + T(x, y, z, t), \tag{2.1a}$$

$$\tilde{S} = S_z z + S(x, y, z, t). \tag{2.1b}$$

The density field is then given by

$$\rho = \rho_0[1-(\alpha T_z - \beta S_z) z - (\alpha T - \beta S)], \tag{2.2}$$

where α and β are the coefficients of expansion for heat and salt, with α and β positive. In order that the density gradient is statically stable we require that

$$\alpha T_z > \beta S_z > 0. \tag{2.3}$$

This also ensures that the temperature gradient is stable and the salinity gradient is unstable.

The dimensionless equations of motion are then

$$\sigma^{-1}(\boldsymbol{u}_t + \boldsymbol{u}\cdot\nabla\boldsymbol{u}) = -\nabla P + (R_T T - R_S S)\,\boldsymbol{k} + \nabla^2\boldsymbol{u}, \tag{2.4a}$$

$$T_t + \boldsymbol{u}\cdot\nabla T + w = \nabla^2 T, \tag{2.4b}$$

$$S_t + \boldsymbol{u}\cdot\nabla S + w = \tau\nabla^2 S, \tag{2.4c}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{2.4d}$$

where we have used a lengthscale l, a timescale l^2/κ_T, a temperature scale lT_z and a salinity scale lS_z. The velocity field is $\boldsymbol{u} = (u, v, w)$, and $\boldsymbol{k}$ is the unit vector in the z-direction. There are four parameters in (2.4), given by

$$\sigma = \frac{\nu}{\kappa_T}, \quad \tau = \frac{\kappa_S}{\kappa_T}, \quad R_T = \frac{\alpha g T_z l^4}{\nu\kappa_T}, \quad R_S = \frac{\beta g S_z l^4}{\nu\kappa_T}, \tag{2.5}$$

where ν is the kinematic viscosity and κ_T and κ_S are the thermal and saline diffusion coefficients. The dynamic pressure p is given by

$$p = \rho_0\{P - gz[1-\tfrac{1}{2}(\alpha T_z - \beta S_z) z]\}. \tag{2.6}$$

We look for a steady z-independent solution to (2.4) which represents motion in salt fingers. A periodic solution of this form (with $u = v = 0$) is

$$w = W_s(x, y) = \hat{W}\sin x\cos dy, \tag{2.7a}$$

$$T = T_s(x, y) = \hat{T}\sin x\cos dy, \tag{2.7b}$$

$$S = S_s(x, y) = \hat{S}\sin x\cos dy, \tag{2.7c}$$

$$P = 0, \tag{2.7d}$$

with

$$\hat{W} = -\hat{T}(1+d^2), \quad \hat{W} = -\tau\hat{S}(1+d^2), \tag{2.8a}$$

and

$$\hat{W}(1+d^2) = R_T\hat{T} - R_S\hat{S}. \tag{2.8b}$$

Eliminating $\hat{W}$, $\hat{T}$ and $\hat{S}$ from (2.8) gives

$$R_S - \tau R_T = \tau(1+d^2)^2. \tag{2.9}$$

This equation defines the lengthscale l. Dimensionally it gives

$$l^4 = \frac{\nu(1+d^2)^2}{\beta g S_z/\kappa_S - \alpha g T_z/\kappa_T}. \tag{2.10}$$

The x-period of the fingers is then $2\pi/l$ and the y-period is $2\pi/dl$. If $d = 1$ then the fingers defined by (2.7) are square. If $d = 0$ we obtain two-dimensional fingers, uniform in the y-direction. If $d \to \infty$ we also obtain two-dimensional fingers uniform in the x-direction, provided the lengthscale is changed to dl.

Experimentally the strength of salt fingers is determined not by measuring vertical velocities, i.e. $\hat{W}$, but by measuring the amount of heat and salt transported by the fingers. The amount of heat transported downwards by the salt fingers is

$$F_T = -\overline{wT}\,\kappa_T T_z,$$

where $(\bar{\ })$ denotes an x-average over period 2π and a y-average over period $2\pi/d$. Thus

$$F_T = -\kappa_T T_z \hat{W}\hat{T}\,\overline{\sin^2 x}\,\overline{\cos^2 dy} = \frac{1}{4}\frac{\kappa_T T_z \hat{W}^2}{1+d^2}. \tag{2.11}$$

The $\frac{1}{4}$ appears as a result of averaging in both the x- and y-directions. In two-dimensions only the x-direction is averaged, so $\frac{1}{2}$ appears instead of $\frac{1}{4}$. The amount of salt transported downwards is given by

$$F_S = \frac{1}{4}\frac{\kappa_T S_z \hat{W}^2}{1+d^2}. \tag{2.12}$$

It can then be shown that

$$\hat{W}^2 = \frac{4gl^4}{\nu\kappa_T(1+d^2)}(\beta F_S - \alpha F_T). \tag{2.13}$$

3. The perturbations

We perturb the salt fingers by putting

$$u = u', \tag{3.1a}$$

$$v = v', \tag{3.1b}$$

$$w = W_s(x,y) + w', \tag{3.1c}$$

$$T = T_s(x,y) + T', \tag{3.1d}$$

$$S = S_s(x,y) + S', \tag{3.1e}$$

where the quantities with a subscript s represent the steady salt-finger field and the primed quantities are the infinitesimal perturbation to the steady state.

In order to eliminate the pressure from the problem, we work with the vorticity equation, obtained by taking the curl of (2.4a). Then the linearized equations for the perturbation quantities are

$$\left(\frac{\partial}{\partial t} - \sigma\nabla^2\right)\boldsymbol{\omega}' + \sigma\boldsymbol{k}\wedge\boldsymbol{\nabla}(R_T T' - R_S S') = \boldsymbol{\nabla}\wedge(\boldsymbol{u}_s\wedge\boldsymbol{\omega}') + \boldsymbol{\nabla}\wedge(\boldsymbol{u}'\wedge\boldsymbol{\omega}_s), \tag{3.2a}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T' + w' = -\boldsymbol{u}'\cdot\boldsymbol{\nabla}T_s - \boldsymbol{u}_s\cdot\boldsymbol{\nabla}T', \tag{3.2b}$$

$$\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)S'+w' = -\boldsymbol{u}'\cdot\nabla S_s-\boldsymbol{u}_s\cdot\nabla S', \tag{3.2c}$$

$$\nabla\cdot\boldsymbol{u}' = 0, \tag{3.2d}$$

where

$$\boldsymbol{\omega}' = \nabla\wedge\boldsymbol{u}'. \tag{3.3}$$

The steady salt-finger solution can be written as

$$\begin{pmatrix} W_s \\ T_s \\ S_s \end{pmatrix} = \mathscr{R}\left\{\begin{pmatrix} \hat{W} \\ \hat{T} \\ \hat{S} \end{pmatrix}\frac{1}{4\mathrm{i}}\{\exp[\mathrm{i}(x+dy)]+\exp[\mathrm{i}(x-dy)] -\exp[-\mathrm{i}(x+dy)]-\exp[-\mathrm{i}(x-dy)]\right\}. \tag{3.4}$$

The coefficients of (3.2) are thus periodic in x and y, and independent of z and t. Hence there will be a Floquet solution to (3.2) in the form of a double sum

$$\begin{pmatrix} \boldsymbol{u}' \\ T' \\ S' \end{pmatrix} = \mathscr{R}\left\{\exp[\mathrm{i}(\gamma x+\delta y+mz+\omega t)]\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\begin{pmatrix} \boldsymbol{u}_{np} \\ T_{np} \\ S_{np} \end{pmatrix}\exp[\mathrm{i}(nx+pdy)]\right\}. \tag{3.5}$$

The symmetry of the equations implies we need only consider $0 \leqslant \gamma \leqslant \frac{1}{2}$, $0 \leqslant \delta \leqslant \frac{1}{2}$ and $n+p$ even.

Substituting the solution (3.5) into the governing equation (3.2) gives five independent equations relating the constants $\boldsymbol{u}_{np}$, T_{np} and S_{np}. The equations are given in the Appendix and are valid for every integer n and p.

In order to consider perturbations at all wavelengths it would be necessary to solve the infinite set of equations (A 1)–(A 5). This would determine $\mathrm{i}\omega$ for given values of σ, τ, R_T, $\hat{W}$, γ, δ and m.

In this paper we consider only long-wavelength perturbations. This is done by using the lowest-order truncation to the equations that couples the salt fingers to the long lengthscale, namely we assume $\boldsymbol{u}_{np}$, T_{np} and S_{np} are each zero if either $|n| > 1$ or $|p| > 1$. This truncation is appropriate when $\mu^2 \equiv \gamma^2+\delta^2+m^2 \ll 1$.

Performing the truncation leaves 25 equations in 25 unknowns, namely u_{np}, v_{np}, w_{np}, T_{np} and S_{np} for $m = 0, 1, -1$ and $p = 0, 1, -1$ and $n+p$ even. The solvability condition for this set of equations then determines $\mathrm{i}\omega$ in terms of σ, τ, R_T, $\hat{W}$, γ, δ and m.

To find $\mathrm{i}\omega$ the set of equations (A 1)–(A 5) is first solved for u_{np}, v_{np}, w_{np}, T_{np} and S_{np} in terms of u_{00}, v_{00}, w_{00}, T_{00} and S_{00}, for the following values of n and p: $n = 1$, $p = 1$; $n = 1$, $p = -1$; $n = -1$, $p = 1$; $n = -1$, $p = -1$. This is done by multiplying (A 1) by $\delta+pd$ and (A 2) by $\gamma+n$ and adding. Using (A 5), this eliminates u_{np} and v_{np}. Then (A 3) and (A 4) can be used to eliminate T_{np} and S_{np}, which leaves an equation that gives w_{np} in terms of u_{00}, v_{00}, w_{00}, S_{00} and T_{00}.

The values for w_{np} and hence u_{np}, v_{np}, T_{np} and S_{np} are then known in terms of the $n = 0, p = 0$ variables. Thus it is possible to substitute these values into the $n = 0$, $p = 0$ equations to obtain five equations in the five unknowns u_{00}, v_{00}, w_{00}, T_{00} and S_{00}. These equations are extremely long and complicated and to get results that can be understood it is necessary to simplify the equations.

4. The instabilities

To proceed further we need to make more assumptions and to look specifically for the different instabilities that we hope to find.

4.1. *Collective instability*

We suppose that γ, δ, m and μ are all the same order and that $\sigma \gg 1$. Simplifying the equations and substituting into (A 1)–(A 5) when $n = p = 0$ gives

$$\frac{\omega^2}{\sigma} = \frac{(\gamma^2+\delta^2)(R_T-R_S)}{\gamma^2+\delta^2+m^2} + O(\mu^2), \tag{4.1}$$

and that the system is unstable if

$$\frac{3m^2(\gamma^2+d^2\delta^2)}{4(\gamma^2+\delta^2)(\gamma^2+\delta^2+m^2)}\hat{W}^2 > \sigma(R_T-R_S). \tag{4.2}$$

The instability is a growing oscillation, oscillating at the internal-wave frequency.

In terms of the fluxes through the fingers, using (2.13), this instability criterion can be written as

$$\frac{\beta F_S-\alpha F_T}{\nu(\alpha T_z-\beta S_z)} > \frac{(1+d^2)(\gamma^2+\delta^2)(\gamma^2+\delta^2+m^2)}{3m^2(\gamma^2+d^2\delta^2)}. \tag{4.3}$$

We write $\gamma = \mu \sin\theta\cos\phi$, $\delta = \mu\sin\theta\sin\phi$ and $m = \mu\cos\theta$, with $\mu = (\gamma^2+\delta^2+m^2)^{\frac{1}{2}}$. Then the instability criterion is

$$A > \frac{1+d^2}{3}\frac{1}{\cos^2\theta\,(\cos^2\phi+d^2\sin^2\phi)}, \tag{4.4}$$

where we have defined a stability parameter

$$A = \frac{\beta F_S-\alpha F_T}{\nu(\alpha T_z-\beta S_z)}. \tag{4.5}$$

The minimum value of the stability parameter occurs when $\theta = 0$ and $\phi = 0$ or $\frac{1}{2}\pi$. Thus the system first becomes unstable to the collective instability when

$$A > \tfrac{1}{3}(1+d^2)\min\{1, 1/d^2\}. \tag{4.6}$$

For salt fingers with a square cross-section, d equals one; hence the system is unstable for

$$A > \tfrac{2}{3}. \tag{4.7}$$

For two-dimensional salt fingers when $d = 0$ or ∞ the system is unstable for

$$A > \tfrac{1}{3}. \tag{4.8}$$

This agrees with the result obtained for the stability parameter in Holyer (1981). Thus square salt fingers are slightly more stable to internal-wave perturbations than two-dimensional salt fingers.

4.2. *Non-oscillatory instability*

A new, non-oscillatory instability was found by Holyer (1984) for which $\gamma = 0$. We look for the three-dimensional version of this instability, assuming also that $\delta = 0$. It is assumed that $m^2 \ll 1$, which is a necessary consequence of the truncation. It can then be shown that there are two separate unstable modes.

(i) If $v_{00} = T_{00} = S_{00} = 0$ and $u_{00} \neq 0$ then

$$i\omega = -\frac{\sigma m^2}{2} \pm \left(\frac{\sigma^2 m^4}{4} + \frac{m^2 \hat{W}^2}{4(1+d^2)}\right)^{\frac{1}{2}}. \tag{4.9}$$

Using (2.13) to write this in terms of fluxes gives

$$i\omega = -\frac{\sigma m^2}{2} \pm \left(\frac{\sigma^2 m^4}{4} + \frac{(\beta F_S - \alpha F_T)\, R_T\, m^2}{(1+d^2)^2\, \alpha T_z}\right)^{\frac{1}{2}}. \tag{4.10}$$

In terms of fluxes through the fingers this is identical with (4.36) in Holyer (1984), if we set $d = 0$. (Looking at (4.9) there is apparently a discrepancy with (4.36). This occurs because the y-average of $\lim_{d\to 0} \cos dy$ equals 1, but the limit as $d \to 0$ of the y-average of $\cos dy$ equals $\frac{1}{2}$. The expressions involving fluxes agree.) Taking the positive sign before the square root, we have a non-oscillatory solution with a positive growth rate. There is instability for all values of d. For square salt fingers, where $d = 1$, the second term in the square root is half of the value that it has at $d = 0$, and the growth rate is smaller than the growth rate for two-dimensional fingers. As $d \to \infty$ this instability disappears, since $i\omega \to 0$.

(ii) If $u_{00} = T_{00} = S_{00} = 0$ and $v_{00} \neq 0$ then the motion in the $n = 0$, $p = 0$ part of the perturbation is in the (y, z)-plane. We find

$$i\omega = -\frac{\sigma m^2}{2} \pm \left(\frac{\sigma^2 m^4}{4} + \frac{m^2 \hat{W}^2}{4(1+1/d^2)}\right)^{\frac{1}{2}}. \tag{4.11}$$

As $d \to \infty$ we get the same growth rate as from (4.9). If $d \to \infty$ the salt fingers are two-dimensional, aligned in the y-direction, so we expect the same growth rate as for two-dimensional fingers aligned in the x-direction. This unstable mode also exists for all d. For $d = 1$ it has the same growth rate as for case (i), and as $d \to 0$, $i\omega \to 0$.

5. Conclusions

It has been shown that the geometry that is chosen for salt fingers, namely two-dimensional or square, does not affect the types of instability to which salt fingers are liable. The collective instability occurs for both geometries. The critical value for the stability parameter $(\beta F_S - \alpha F_T)/\nu(\alpha T_z - \beta S_z)$ changes from $\frac{1}{3}$ for two-dimensional fingers to $\frac{2}{3}$ for square fingers. The non-oscillatory instability found in Holyer (1984) exists for square fingers, as well as two-dimensional ones, though with a slightly smaller growth rate.

The main result of this paper is that qualitatively correct physical processes can be found from a study of two-dimensional salt fingers and that processes, such as the collective instability, will occur for both square and planar salt fingers. The effect of shear on salt fingers (Linden 1974) is to align them with the direction of shear, and so make them two-dimensional. Since the stability parameter is smaller for planar fingers than for square ones, it may be expected that salt fingers are more prone to the collective instability in shear flows.

The experiments of Stern & Turner (1969) are most relevant to the collective instability theory presented here. They used salt and sugar for their experiment, with a deep layer of salt-stratified water beneath a uniform upper layer of lower-salinity water that contained sugar. Salt fingers appear at the interface as soon as the two layers are formed, and they penetrate into the lower, stratified, fluid. As the salinity gradient is reduced, longer fingers form, which break up, apparently by the collective instability mechanism, to give way to a well-stirred, convective layer, which is maintained by the flux through a thin salt-finger layer at the interface. In these experiments the stability parameter A was shown to be the parameter grouping that determined the stability of the system, and marginal stability occurred when it lay between 1.2 and 2.8. It would be helpful to the theoretical work to have more experiments carried out in similar systems where there is a linear heat and salt stratification, in order to find out if the same results are found in a heat–salt system and to study the destruction of long fingers in more detail.

The majority of recent experiments (e.g. McDougall & Taylor 1984) have considered fluxes through an interface at which salt fingers are present. One may doubt whether the infinite model of this paper is relevant to these experiments, however: since the finger width is small compared with the height of the interfacial layer, the central part of the interface should be unaffected by the ends of the fingers, and Stern (1969) has argued that such interfaces should be marginally stable to the collective instability. Values of the stability parameter varying between 0.002 and 5 have been found for different configurations in heat–salt and salt–sugar systems. It is possible that some of the small values for the stability parameter occur because the non-oscillatory instability is the important mechanism, and not the collective instability, in the breakdown of short fingers. The shorter-wavelength non-oscillatory instability is likely to be responsible for the bulges and other small-scale irregularities that can be seen on salt fingers. The main thing that is needed from the experiments that is not currently available is good accurate profiles of the temperature and salinity within the salt finger interface.

The most pressing theoretical problem is the study of the non-oscillatory instability and its weakly nonlinear development. It starts to grow at the region of maximum shear in the salt fingers. As it grows it could either tend to a stable cat's-eye type of solution, or the recirculating regions of the linear theory could amalgamate and completely disrupt the salt-finger field. Experimentally, it would help if this instability could be separated from the collective instability, and this will only be possible when the non-oscillatory instability is better understood.

Numerically, the most realistic calculations that have been carried out were made by Piaczek & Toomre (1980). They obtained numerical solutions that modelled the growth of two-dimensional salt fingers at an interface. It should be possible, with the increased storage and speed of computers, to use an amplitude expansion or a spectral method to look at the strongly nonlinear process of the breakdown of long salt fingers into interfaces separated by layers. Such a calculation would probably have to be restricted to two-dimensional equations, but this paper supports the idea that the results from two-dimensional and three-dimensional calculations would be qualitatively the same.

I would like to thank one of the referees of a previous paper (Holyer 1984) for inspiring me to do this lengthy calculation.

Appendix

The equations that determine the coefficients $\boldsymbol{u}_{np}$, T_{np} and S_{np} are given below. We define

$$\mu_{np}^2 = (\gamma+n)^2+(\delta+pd)^2+m^2.$$

The component of the vorticity equation in the x-direction is

$$\begin{aligned}(\mathrm{i}\omega+\sigma\mu_{np}^2)\,[(\delta+pd)\,w_{np}-mv_{np}]-\sigma(\delta+pd)\,(R_T\,T_{np}-R_S\,S_{np})\\ = -\tfrac{1}{4}m\hat{W}\{[(\delta+(p-1)\,d)]\,(w_{n-1,\,p-1}-w_{n+1,\,p-1})\\ +[\delta+(p+1)\,d]\,(w_{n-1,\,p+1}-w_{n+1,\,p+1})\}\\ -\tfrac{1}{4}(d^2-m^2)\,\hat{W}(v_{n-1,\,p-1}+v_{n-1,\,p+1}-v_{n+1,\,p-1}-v_{n+1,\,p+1})\\ +\tfrac{1}{4}d\hat{W}[(\gamma+n-1)\,(u_{n-1,\,p-1}-u_{n-1,\,p+1})+(\gamma+n+1)\,(u_{n+1,\,p+1}-u_{n+1,\,p-1})]\\ -\tfrac{1}{4}(\delta+pd)\,\hat{W}(u_{n+1,\,p+1}-u_{n-1,\,p+1}+u_{n+1,\,p-1}+u_{n-1,\,p-1}).\end{aligned} \tag{A 1}$$

The component of the vorticity equation in the y-direction is

$$\begin{aligned}(\mathrm{i}\omega+\sigma\mu_{np}^2)\,[mu_{np}-(\gamma+n)\,w_{np}]+\sigma(\gamma+n)\,(R_T\,T_{np}-R_S\,S_{np})\\ = \tfrac{1}{4}m\hat{W}[(\gamma+n-1)\,(w_{n-1,\,p-1}+w_{n-1,\,p+1})\\ -(\gamma+n+1)\,(w_{n+1,\,p-1}+w_{n+1,\,p+1})]\\ +\tfrac{1}{4}(1-m^2)\,\hat{W}(u_{n-1,\,p-1}+u_{n-1,\,p+1}-u_{n+1,\,p-1}-u_{n+1,\,p+1})\\ -\tfrac{1}{4}\hat{W}\{[(\delta+(p+1)\,d]\,(v_{n+1,\,p+1}+v_{n-1,\,p+1})+[\delta+(p-1)\,d]\,(v_{n-1,\,p-1}+v_{n+1,\,p-1})\}\\ +\tfrac{1}{4}(\gamma+n)\,d\hat{W}(v_{n-1,\,p-1}-v_{n-1,\,p+1}-v_{n+1,\,p-1}+v_{n+1,\,p+1}).\end{aligned} \tag{A 2}$$

The temperature equation is

$$\begin{aligned}(\mathrm{i}\omega+\mu_{np}^2)\,T_{np}+w_{np} = \frac{\hat{W}}{4(1+d^2)}\,[u_{n+1,\,p+1}+u_{n+1,\,p-1}\\ +u_{n-1,\,p+1}+u_{n-1,\,p-1}+d(v_{n+1,\,p+1}-v_{n+1,\,p-1}-v_{n-1,\,p+1}+v_{n-1,\,p-1})]\\ -\tfrac{1}{4}m\hat{W}(T_{n-1,\,p-1}+T_{n-1,\,p+1}-T_{n+1,\,p-1}-T_{n+1,\,p+1}).\end{aligned} \tag{A 3}$$

The salt equation is

$$\begin{aligned}(\mathrm{i}\omega+\tau\mu_{np}^2)\,S_{np}+w_{np} = \frac{\hat{W}}{4\tau(1+d^2)}\,[u_{n+1,\,p+1}+u_{n+1,\,p-1}\\ +u_{n-1,\,p+1}+u_{n-1,\,p-1}+d(v_{n+1,\,p+1}-v_{n+1,\,p-1}-v_{n-1,\,p+1}+v_{n-1,\,p-1})]\\ -\tfrac{1}{4}m\hat{W}(S_{n-1,\,p-1}+S_{n-1,\,p+1}-S_{n+1,\,p-1}-S_{n+1,\,p+1}).\end{aligned} \tag{A 4}$$

Finally, the continuity equation is

$$(n+\gamma)\,u_{np}+(\delta+pd)\,v_{np}+mw_{np} = 0. \tag{A 5}$$

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